Note Title 5/1/2018

DERIVATIVES OF A^x AND $\log_a x$

RECALL: $a^{\log_a x} = x$ $\qquad x\ln a = \ln a^x$

$\therefore e^{\ln a} = a$

$\therefore e^{\ln a^x} = a^x$

$\therefore e^{x\ln a} = a^x$ $\qquad \frac{d}{dx}\, \underline{x\ln a}$

$\therefore \frac{d}{dx} a^x = \frac{d}{dx} e^{x\ln a}$ $\qquad 1\ln a + (0)(x)$

$= e^{x\ln a} \cdot \ln a$

$= \cancel{e}^{\ln a^x} \cdot \ln a = a^x \ln a$

$\therefore \frac{d}{dx} a^{f(x)} = e^{f(x)\ln a} \cdot \ln a \cdot f'(x)$

IE Find $\frac{dy}{dx}$ if $y = 5^x$

SOLN

$$\frac{dy}{dx} = e^{x \ln 5} \cdot \ln 5$$

IE $y = 5^{(x^2+2x)}$ $\quad \frac{dy}{dx} = e^{(x^2+2x)\ln 5} \cdot \ln 5 (2x+2)$

$$= 5^{(x^2+2x)} \ln 5 (2x+2)$$

IE $y = 4^{x^4}$ $\quad \frac{dy}{dx} = e^{x^4 \ln 4} \cdot \ln 4 \cdot 4x^3$

$$= 4^{x^4} \cdot \ln 4 (4x^3)$$

$\log_a x$

RECALL: $\log_a x = \frac{\log x}{\log a} = \frac{\log_e x}{\log_e a} = \frac{\ln x}{\ln a}$

$$\therefore \frac{d}{dx} \log_a x = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right)$$

$$= \frac{1}{\ln a} \frac{d}{dx} (\ln x)$$

$$= \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$$

$$\frac{d}{dx} \log_a f(x) = \frac{d}{dx} \frac{\ln f(x)}{\ln a}$$

$$= \frac{1}{\ln a} \cdot \frac{d}{dx} \ln f(x)$$

$$= \frac{1}{\ln a} \cdot \frac{1}{f(x)} \cdot f'(x)$$

$$\frac{d}{dx} \log_a f(x) = \frac{f'(x)}{\ln a \, f(x)}$$

Ex: IF $y = \log_2 x$ $\quad \frac{dy}{dx} = \frac{1}{x \ln 2}$

Ex: $y = \log_{73} x$ $\quad \frac{dy}{dx} = \frac{1}{x \ln 73}$

IE $y = \log_5 (x^2 + 6x)$

$$\frac{dy}{dx} = \frac{1}{\ln 5} \cdot \frac{1}{x^2+6x} \cdot 2x+6$$

$$\frac{dy}{dx} = \frac{2x+6}{\ln 5\,(x^2+6x)}$$

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